Self-Regulating Shear Flow Turbulence: A Paradigm for the L to H Transition

P. H. Diamond* and Y.-M. Liang
University of California, San Diego, La Jolla, California 92093-0319

B. A. Carreras
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830-8070

P. W. Terry
University of Wisconsin, Madison, Wisconsin 53706
(Received 23 July 1993)

A self-consistent model of the L to H transition is derived from coupled nonlinear envelope equations for the fluctuation level and radial electric field shear E_r'. These equations exhibit a supercritical bifurcation between dual L-mode and H-mode fixed points. The transition occurs when the turbulence level is large enough for the Reynolds stress drive to overcome the damping of the $\mathbf{E}\times\mathbf{B}$ flow. This defines a power threshold for the transition, which is calculated and found to be consistent with experimental findings.

PACS numbers: 52.55.Hc

Auxiliary heated tokamak experiments in the L-mode confinement regime reveal a degradation of confinement with increasing power. In this confinement mode, the energy confinement time decreases as the square root of the power input and increases linearly with the plasma current [1]. In 1982, the H mode was observed in the ASDEX tokamak [2], and has since been reproduced in all significant tokamaks and stellarators. The H-mode confinement regime is characterized by an increased factor of 2 to 3 in the confinement time, a decrease of the plasma edge fluctuations, and the formation of a transport barrier at the plasma edge (2 to 4 cm from the limiter/separatrix) with plasma density n developing a steep gradient in this region. The transition from the L to H mode is rapid (less than 2 msec) and happens for input powers above a threshold P_{th}. This threshold scales as $P_{\text{th}} \propto B_T n$, where B_T is the toroidal magnetic field [3]. The transition seems to be independent of heating method and magnetic field geometry [4].

The transport barrier is due to the formation of an edge E_r' layer. The region of strong shear correlates well with the steep density gradient region and the fluctuation suppression [5]. The L to H transition and the E_r' layer formation are accompanied by the onset of an average poloidal flow in the edge layer. Experimentally, it has not yet been possible to determine the causal relation of the E_r' layer formation and the onset of the L to H transition.

An even higher confinement mode, the VH mode [6], was discovered recently in the DIII-D tokamak. Improving the wall conditioning with boronization depresses the edge density and radially broadens the good confinement region up to 60% of the minor radius. It is quite likely that the VH mode regime is a natural extension of the L to H transition. The discovery of the VH mode is of special significance since it suggests a link of fluctuations and transport dynamics to the improved confinement transition which cannot be described only by orbit loss mechanisms operative within ρ_θ of the edge.

There are several theoretical models for the L to H transition based on sheared electric field effects on turbulence, all based on the enhanced eddy decorrelation induced by the shear electric field with the consequent turbulence suppression [7]. After the causal relation between the radial electric field and the L to H transition was first suggested [8], it was followed by more detailed models [9,10] based on a two step process: (1) At a certain level of injected power in a tokamak, the edge particle losses or poloidal asymmetry create or modify the radial electric field and (2) this electric field is then responsible for the reduction of the fluctuation level and transport. However, since it has been shown that turbulence can modify the profile of average flows through Reynolds stress [11,12], the electric field profile and turbulence must be calculated self-consistently.

Here, we present a bifurcation theory model for the L to H transition, the basic components of which are addressed elsewhere [7,13]. We propose a self-consistent model of the transition derived from coupled nonlinear envelope equations for the fluctuation level and E_r'. A broad class of detailed turbulence models (parallel flow gradient drive, drift wave turbulence, resistive interchange, etc.) can be simplified to such an envelope equation form. We approach this goal by examining the evolution of E_r' and the mean square fluctuation for a single helicity system. Note that the radial scale of such a system corresponds to the width of a standard H-mode shear layer (1 to 2 cm). The model derived here is a paradigm.

First, let us consider the fluctuation evolution equation. The nonlinear growth rate in the presence of E_r' is the result of balancing the linear drive γ_0 with the damping caused by the nonlinear coupling to other helicities and the E_r' fluctuation suppression effect. The effect of the energy leakage to other helicities can be represented by an amplitude dependent diffusivity, $D_k = \rho_s^2 c_s^2 \sum_{k'} k_\theta'^2 |\tilde{n}_{k'}/$

0031-9007/94/72(16)/2565(4)$06.00

$n_0|^2\Gamma_{k-k'}$, where $c_s=\sqrt{T_{e0}/m_i}$ is the speed of sound, $\rho_s=c_s/\Omega_i$ is the sound Larmor radius, k_θ is the poloidal wave number, and $\Gamma_{k-k'}$ is the effective correlation time. The resulting damping rate is $-D_k/W_k^2$, with W_k the radial width of the instability. An average shear flow introduces a symmetry-breaking term, $k_\theta V_E' x$ (odd parity in relation to the resonance surface position) into the dynamical equations. Since the equations are invariant under the transformation $x\to -x$, $V_E'\to -V_E'$, the damping must be a function of $V_E'^2$. The characteristic symmetry-breaking parameter is $\Omega\equiv k_\theta V_E' W_k/\gamma_0$; therefore, the flow shear damping rate is proportional to $-\gamma_0\Omega^2$. The proportionality coefficient depends on the particular instability. Therefore, the nonlinear growth rate has the general form $\gamma_{\rm NL}=\gamma_0-D_k/W_k^2-\gamma_0\Omega^2$. Defining the density fluctuation level $E\equiv|\tilde n_k/n_0|^2$, the equation for the fluctuation evolution is

$$\frac{1}{2}\frac{dE}{dt}=\gamma_0 E-\alpha_1 E^2-\alpha_2 UE\,, \tag{1}$$

where $U\equiv\langle V_E'\rangle^2$, and the angular brackets, $\langle\,\rangle$, indicate the poloidal and toroidal angle average over a magnetic flux surface. For different turbulence models, the coefficients α_1 and α_2 are given in Table I.

The average poloidal flow profile can be modified by the plasma turbulence via the Reynolds stress [11,12]. This mechanism has been invoked to explain the differential rotation of the solar atmosphere [14]. The poloidal flow profile evolution equation is derived by taking the flux surface average of the momentum balance equation:

$$\frac{\partial\langle V_\theta\rangle}{\partial t}=-\frac{\partial}{\partial r}\left[\langle\tilde V_r\tilde V_\theta\rangle-\frac{1}{\rho_m\mu_0}\langle\tilde B_r\tilde B_\theta\rangle\right]-\mu\langle V_\theta\rangle\,. \tag{2}$$

The first term in the right hand side (rhs) is the Reynolds stress and the second term is the damping caused by magnetic pumping. For the Reynolds stress to have a nonzero contribution requires radial wave propagation and the radial inhomogeneity of the turbulence, i.e., via $\langle V_\theta'\rangle$ tilting the eddies [15]. The nonuniformity of the turbulence results from edge effects and from the presence of low rational surfaces [16]. From the ion momentum balance equation, the average poloidal flow is directly related to the E_r, $E_r/B=-\langle V_\theta\rangle+(\rho_s c_s/P_i)(dP_i/dr)$. For simplicity, and since inhomogeneities in the wave energy flux induce the smallest scale variation in E_r' [11,12] via Reynolds stress effects on $\langle V_\theta\rangle'$, we neglect the ion pressure. The effect of this term has been included in a more detailed model of drift wave turbulence bifurcation dynamics [17].

TABLE I. Coefficients of Eqs. (1) and (3) for different turbulence models.

	Parallel flow gradient driven instability	Resistive interchange instability	Drift thermal instability
α_1	$\dfrac{\gamma_0\rho_s^4 c_s^2}{W_0^8\vert V_\parallel'\vert^2}$	$\dfrac{k_\theta^2\rho_s^2c_s^2}{\gamma_0 W_0^6\Lambda^{5/3}}$	0
α_2	$\dfrac{k_\theta^2W_0^2}{2\gamma_0}\dfrac{W_0^2}{\rho_s^2}$	$\dfrac{k_\theta^2W_0^2}{\gamma_0}$	$\dfrac{k_\theta^2W_0^2}{\omega_a}$
α_3	$\dfrac{k_\theta^2c_s^2}{2\gamma_0}$	$\dfrac{k_\theta^2\rho_s^2c_s^2}{\gamma_0W_0^6\Lambda^{2/3}}$	$\dfrac{k_\theta^2\rho_s^2c_s^2}{\omega_aW_0^6}$
W_0	$\left(\dfrac{\gamma_0\rho_sL_s}{k_\theta c_s}\right)^{1/2}$	$\left(\dfrac{\gamma_0^2r_cL_n}{k_\theta^2c_s^2}\right)^{1/2}$	$\left(\dfrac{\chi_\perp L_s^2}{\chi_\parallel k_\theta^2}\right)^{1/4}$
$\dfrac{b}{a}$	$\left(\dfrac{L_g}{L_s}\right)^2\dfrac{2\mu L_s}{k_\theta\rho_sc_s}$	$\left(\dfrac{k_\theta^2}{\langle k_\theta^2\rangle}\right)^{1/2}$	0

Taking the radial derivatives of Eq. (2), it becomes the evolution equation for $\langle V_E'\rangle$. To close the system of equations, we have to evaluate $\partial^2\langle\tilde V_r\tilde V_\theta\rangle/\partial r^2$ in terms of the density fluctuations. This term is quadratic in the fluctuations, and hence proportional to E. It contains the factor $c_s^2\rho_s^2$ because of the normalization. It is also proportional to the symmetry-breaking term Ω (zero in the symmetric limit), and to $-k_\theta/W^3$ because of the four derivatives involved. Therefore, $\partial^2\langle\tilde V_r\tilde V_\theta\rangle/\partial r^2\approx -k_\theta c_s^2\rho_s^2\Omega E/W^3$. After multiplying the whole equation by $\langle V_E'\rangle$, Eq. (2) becomes (note $U=\langle V_E\rangle'^2$)

$$\frac{1}{2}\frac{dU}{dt}=-\mu U+\alpha_3 UE\,. \tag{3}$$

The coefficient α_3 is given in Table I. The three α coefficients are not independent; conservation of energy requires $\alpha_2=\alpha_3W^4/c_s^2\rho_s^2$. Equation (3) shows that the radial symmetry-breaking effect induced by $\langle V_E'\rangle$ leads to an amplification of the flow shear. This is a dynamo type instability, and is similar to the anisotropic kinetic alpha instability [18]. The first term in the rhs is the magnetic pumping damping term. Note that energy conservation is explicitly accounted for in that turbulence energy lost by E_r' damping is converted to V_E' drive via the Reynolds stress. The net effect of this is to "channel" input energy between an E_r' branch and a fluctuation branch.

Equations (1) and (3) constitute the paradigmatic model for the L to H transition. They are similar to the Verhulst population model [19]. The flow shear is analogous to the predator species and the fluctuation level to the prey species. By conveniently normalizing the time, fluctuation level, and shear flow ($\tau\equiv\gamma_0 t$, $\bar E\equiv\alpha_1E/\gamma_0$, and $\bar U\equiv U\alpha_2$), one can show that the model depends only on two dimensionless parameters, $a=\alpha_3/\alpha_1$ and $b=\mu/\gamma_0$. In discussing this model, the instability growth rate γ_0 will be used as a control parameter. Apart from the trivial equilibrium solution $E=U=0$, Eqs. (1) and (4) have two fixed points: (1) $E=\gamma_0/\alpha_1$ and $U=0$ and (2) $E=\mu/\alpha_3$, $U=(\gamma_0-\alpha_1\mu/\alpha_3)/\alpha_2$. The first, L-mode type, is stable for $\gamma_0<\alpha_1\mu/\alpha_3$ and the second, H-mode type, is stable for $\gamma_0>\alpha_1\mu/\alpha_3$. The transition from L to H happens when the instability drive is large enough so that $\gamma_0=\alpha_1\mu/\alpha_3$. At that point, the L-mode root goes unstable, allowing transition to the H regime. From the fixed point solution, it is clear that for $\gamma_0 < \alpha_1\mu/\alpha_3$, all instability free energy goes into fluctuations and the flow damping is strong enough to avoid any flow generation. In contrast, for $\gamma_0 > \alpha_1\mu/\alpha_3$, most of the instability free energy goes to build up the poloidal flow and the fluctuation level is low, thus reconciling decreased fluctuation levels with increased free energy input, γ_0. In the second case, although the free energy available is larger than for the first fixed point, the fluctuation level is lower. Note also that in the H-mode type solution, the fluctuation level is controlled by μ while the shear flow is controlled by γ_0. This is contrary to standard intuition, where $E \sim \gamma_0$. Notice, therefore, that the H mode is a marginal state to a flow shear dynamo instability. In this state, E_r' can adjust easily to maintain nonlinear saturation of fluctuations in the presence of increased γ_0. Also, $U = \langle V_\theta' \rangle^2$, the second fixed point corresponds to two physical solutions, each with a different sign for the flow. In this model, each of these solutions has equal probability for being accessed, albeit with other effects such as ∇P_i and orbit loss that may seed the $E_r' < 0$ solution. Finally, we remind the reader that here "fixed points" are fixed relative to slow, transport time scales.

In Fig. 1, we have plotted the solution of Eqs. (1) and (3) for $b=1$, and for four different values of a. For $a=0.5$ [Fig. 1(a)], the fluctuation grows to the L-mode saturation level, $\bar{E}=1$, and no flow is generated. By increasing a to 1.3 [Fig. 1(b)], the instability saturates first at the L-mode level while later on there is a smooth transition to the H mode, with generation of flow. For higher values of a, the transition is not a smooth decay but has an oscillating component. This oscillating component increases with increasing a in such a way that, for $a \gg b$, the solution does not go through an L-mode phase but goes directly to the H mode via strong relaxation oscillations.

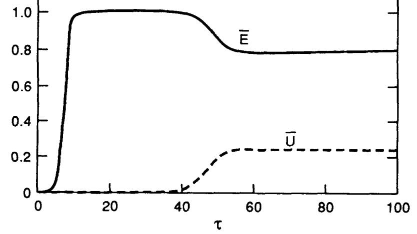

FIG. 1. Time evolution of $\bar{E}$ (continuous line) and $\bar{U}$ (broken line) for $b=1$ and (a) $a=0.5$ and (b) $a=1.3$.

The transition condition, $\gamma_0 > \alpha_1\mu/\alpha_3$, implies that the net energy input rate exceeds the flow damping rate, allowing energy storage in the shear flow. This translates to a power threshold for the H-mode confinement. For a generic drift the wave model, the growth rate is proportional to the temperature gradient, $\gamma_0 \approx \overline{k_\theta \rho_s c_s}(\nabla T/T)$. Using the power balance at the plasma edge, the temperature gradient can be directly related to the power input density, $\nabla T/T \sim aP_{\rm in}/nT\chi$, where χ is the edge perpendicular heat diffusivity. The bifurcation condition leads to the power threshold $P_{\rm th} \sim \mu nT(aR/\alpha_3)\sqrt{T/M}$. For generic drift wave-type turbulence $\alpha_3 = c_z/L_s$, so $P_{\rm th} \cong (\mu nT)_{\rm edge} aRL_s$. Here $(\mu nT)_{\rm edge}$ is to be evaluated using L-mode parameters, just prior to transition (i.e., μ should be evaluated for plateau on Pfirsch-Schluter conditions). Note that $P_{\rm th}$ scales (unfavorably) with surface area and with density. Also, taking $T \sim B_T$ (also valid for generic drift wave models) yields a scaling in accord with many experiments. It should also be noted that a related exercise of identifying a "dimensionless parameter" for the $L \to H$ transition yields $\rho = (V_{Ti}/L_T\mu)(L_n/L_s) \times (\omega/L_n)^2 \sim T_i^3/n$, also in accord with experimental findings. The ratio between the H- and L-mode fluctuation levels, $b/a = \alpha_1\mu/\alpha_3\gamma_0$, depends on the dynamical model of turbulence. The confinement improvement factor H is directly related to the b/a ratio, $H = (\alpha_1\mu/\alpha_3\gamma_0)^\nu$, with $\nu = \frac{1}{2}$ for strong turbulence. This factor is far from constant. Naturally H is the local edge factor and does not necessarily give the overall confinement improvement. Note also that the model predicts a modest drop in fluctuation levels rather than total suppression.

Equations (1) and (3) satisfy the Kolmogorov theorem; therefore, their solutions are either a stable focus or a nonlinearly stable limit cycle. A local stability analysis can be performed by linearizing the model equations around the two fixed points. There are two roots to the stability equation; one is associated with the fluctuation and the other with the flow. For the L-mode state, the two roots correspond to damped modes, with decay rates $\gamma = -(\mu - \alpha_3\gamma_0/\alpha_1)$ and $\gamma = -\gamma_0$. For the H-mode state and away from the critical point, the modes are oscillatory and damped. In the neighborhood of the critical point [Fig. 1(b)], the modes have zero frequency, and the damping rate is low for the flow and high for the fluctuations. Therefore, the transition is smooth near the critical point and consistent with the smooth, supercritical bifurcation characteristic of the "dithering mode." Since the basic instability rates are different for the two fixed points, there is a symmetry between the L to H and the H to L transitions. The solutions of Eqs. (1) and (3) are shown in Fig. 2 for a case in which b has been changed abruptly. For this solution, b has been decreased from $b=1.7$ to $b=0.6$ at $t=50$ and decreased back to its initial value at $t=150$. In this way, both transitions have been triggered in the calculation. We can see that the L

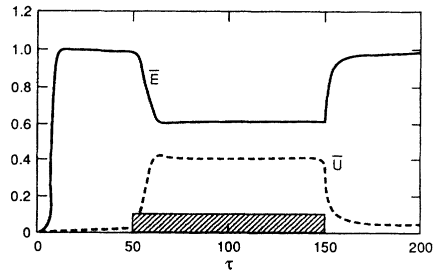

FIG. 2. Time evolution of $\bar{E}$ (continuous line) and $\bar{U}$ (broken line). At $t=0$, $b=1.7$ and $a=1$; in the shaded region, the constant b has been increased to 0.6 to trigger the transitions.

to H transition is slower than the corresponding one from H to L. The transition time from L to H is

$$\tau_t \approx \alpha_1 \ln[(\gamma_0\alpha_3 - \alpha_1\mu)/\alpha_3]/(\alpha_3\gamma_0 - \mu\alpha_1)\,,$$

and is singular at the critical power.

Since in the neighborhood of the critical point the damping rate for the flow is low and for the fluctuations high, we can assume that the fluctuations are slaved to the flow so that the dynamical model can be reduced to a single equation by averaging over fast fluctuation modes. This equation is equivalent to the Landau model for a second-order phase transition. We can use this model to understand what effect an external torque has on the flow parameter near the transition point. The mean flow shear equation with an external torque is

$$\frac{d\langle V_E'\rangle}{dt} = (\alpha_3\gamma_0/\alpha_1 - \mu)\langle V_E'\rangle - \alpha_3\alpha_2/\alpha_1\langle V_E'\rangle^3 + T'_{\rm ext}\,. \qquad (4)$$

The susceptibility of the shear flow to an external torque can be calculated from Eq. (4). In the L-mode phase, $(\partial\langle V_E'\rangle/\partial T'_{\rm ext}) = (\mu - \alpha_3\gamma_0/\alpha_1)^{-1}$, and in the H-mode phase $(\partial\langle V_E'\rangle/\partial T'_{\rm ext}) = \frac{1}{2}(\alpha_3\gamma_0/\alpha_1 - \mu)^{-1}$. The linear susceptibility exhibits a divergence and discontinuity at the transition. If the external torque is large enough to exceed the self-generated shear, it can drive the H-mode phase. The critical value required is $T'_{\rm ext} \geq \frac{1}{2}|\alpha_3\gamma_0/\alpha_1 - \mu|^{3/2}/(\alpha_3\alpha_2/\alpha_1)^{1/2}$. Thus, deep in the L mode, unrealistic torques may be required to affect the shear. Near the transition, the state rapidly transits to a saturated regime where $E_r' \propto (T'_{\rm ext})^{1/3}$ due to nonlinear response effects.

The results of the model presented here suggest that the L to H transition is a second-order phase transition [20]. The poloidal flow gradient is the order parameter. The L mode, characterized by randomly distributed turbulent eddies, is the disordered state. The H mode, characterized by the global shear flow dominating over random convection, is the ordered state. Because the model depends on the square of the shear flow, there is equal probability of a steady state with either sign for the shear flow. Therefore, whatever sign is chosen implies a spontaneous symmetry-breaking effect. Since γ_0 is the control parameter, it plays the role of the temperature in a phase transition, and $\alpha_1\mu/\alpha_3$ is the analog of the critical temperature. Finally, it should be mentioned that a subcritical bifurcation is also a possible route to the L to H transition, the signature of which would be an abrupt discontinuity in order parameter at the threshold. Thus, slowly ramped power scans crossing $P_{\rm crit}$ are necessary to experimentally elucidate the transition mechanism.

The authors gratefully acknowledge fruitful discussions with K. Burrell, A. V. Gruzinov, V. B. Lebedev, H. Levine, M. N . Rosenbluth, C. M. Surko, and F. Wagner. This work is supported by the Department of Energy through Grant No. DE-FG03-88ER-53275 and under Contract No. DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

*Also at General Atomics, San Diego, CA 92186.

[1] D. W. Swain, Nucl. Fusion **21**, 1409 (1981).
[2] F. Wagner *et al.*, Phys. Rev. Lett. **49**, 1408 (1982).
[3] R. J. Groebner, K. H. Burrell, and R. P. Seraydarian, Phys. Rev. Lett. **64**, 3015 (1990).
[4] ASDEX team, Nucl. Fusion **29**, 1959 (1989).
[5] E. J. Doyle *et al.*, Phys. Fluids B **3**, 2300 (1991).
[6] T. H. Osborne *et al.*, Phys. Fluids (to be published).
[7] H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B **2**, 1 (1990).
[8] S. I. Itoh and K. Itoh, Phys. Rev. Lett. **60**, 2276 (1988).
[9] K. C. Shaing, W. A. Houlberg, and E. C. Crume, Comments Plasma Phys. Controlled Fusion **12**, 69 (1988); K. C. Shaing and E. C. Crume, Phys. Rev. Lett. **63**, 2369 (1989).
[10] A. B. Hassam *et al.*, Phys. Rev. Lett. **66**, 321 (1991).
[11] P. H. Diamond and Y. B. Kim, Phys. Fluids B **3**, 1626 (1991).
[12] B. A. Carreras, L. Garcia, and V. E. Lynch, Phys. Fluids B **3**, 1438 (1991).
[13] P. H. Diamond *et al.*, in Proceedings of the 14th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Wurzburg (IAEA, Vienna, 1992).
[14] G. Rüdiger, *Differential Rotation and Stellar Convection* (Gordon and Breach, New York, 1989).
[15] J. F. Drake *et al.*, Phys. Fluids B **4**, 488 (1992).
[16] H. Biglari *et al.*, in Proceedings of the 12th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice (IAEA, Vienna, 1988).
[17] Y.-M. Liang, P. H. Diamond, and B. A. Carreras (to be published).
[18] B. Galanti and P.-L. Sulen, Phys Fluids A **3**, 1778 (1991).
[19] P. F. Verhulst, Nuov. Mem. Acad. Roy. Bruxelles **18**, 1 (1845).
[20] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics* (Pergamon, New York, 1980), Pt. 1, p. 446.